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ORIGINAL RESEARCH

Notation, Redundancy and Fundamental Logical Structure

Carlos Romero[1]

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Abstract

A hundred years ago, Ramsey (*Aristotelian Society Supplementary* 7(1):153-170, 1927) devised a new system of notation to more perspicuously represent the idea that 'p' and '$\neg\neg p$' express the same fact. Recently, Monroy Pérez (*Mind* 131(524):1083–1107, 2022) has argued that we can apply Ramsey's insight to solve a recalcitrant problem for logical elitism—the thesis, most famously defended by Sider (*Writing the Book of the World*, Oxford University Press), that the world's fundamental metaphysical structure includes logical structure. Here, I discuss the general role of language in the framing of logical elitism and present four arguments that show that the Ramseyan strategy for defending logical elitism fails. First, it does not eliminate all arbitrariness even for zeroth-order classical logic (§3). Second, it is a case of a more general strategy that exploits the fact that classical negation is an *involution*; then, choosing the Ramseyan proposal among the other cases is arbitrary (§4). Third, there are proposals to solve the riddle that are *simpler* than every case of the involutory strategy (§5). The discussion has consequences for logical elitism at large: in my fourth argument I contend, contrary to a fairly favoured opinion, that logical elitism should not be thought of as a thesis about *languages*, but as a thesis about *joints* in metaphysical structure (§6).

1 Introduction

Logical elitism is the thesis that the world's fundamental metaphysical structure includes logical structure. One may distinguish between *ontological* and *ideological* versions of it (*cf*. McSweeney, 2019, p. 120). On the ontological variety, terms for logical particles refer to ontologically fundamental *entities*. On the ideological version, terms for logical particles do not refer at all—they are syncategorematic—but they are pieces of ideology that somehow get to represent some of the fundamental 'joints of nature', those aspects giving the fundamental structure of the world. The best developed way to formally present this thesis is by way of Sider's (2011) structural operator, '$\mathcal{S}$', so that, for example, '$\mathcal{S}(\vee)$' says that disjunction marks a joint of nature.

✉ Carlos Romero
carlos.a.romero.c@icloud.com

1 Philosophy, PUCV, Viña del Mar, Chile

Logical elitism faces *the riddle of redundancy*. For our purposes, redundancy can be understood as *definability*: a set of logical particles is redundant if at least one of its elements is definable in terms of some of the others (given a fixed background logic); it is non-redundant otherwise.[1] It is widely held that the fundamental logical particles—if such there be—should not form a redundant set. Now, for our purposes, a set of logical particles is *arbitrary* if it contains a particle X but not a particle Y, even though there is no reason to prefer the former over the latter. The problem comes when, in order to avoid redundancy, we fall into arbitrariness. We avoid redundancy by taking certain definable logical particles out of our set; however, taking X out rather than Y is unmotivated when either can play equivalent theoretical roles—which they can, as we are assuming that they are inter-definable. In such a case, we fall into arbitrariness. It is, as well, widely held that the set of fundamental logical particles should not be arbitrarily formed. Then, 'The riddle of redundancy is that, under elitism, we are compelled to choose among two theoretical vices: arbitrariness and redundancy' (Monroy Pérez, 2022, p. 1084). I agree that redundancy and arbitrariness are both theoretical vices, and I will assume so in what follows.[2]

There are different ways in which one may try to solve or dissolve the riddle. Some prefer to reject the view that redundancy and arbitrariness are both theoretical vices (*cf.* Donaldson 2015, pp. 1072–1073, and Torza 2021); some think that there *is* some non-arbitrary reason why a theory containing one particular set of logical particles, rather than another, is the fundamentally true one, but that it is our fate to remain ignorant of such a reason (*cf.* McSweeney, 2019; Sider, 2020, pp. 209-210); some believe that, although it is determinately true that there is *some* non-redundant set of fundamental logical particles, it is metaphysically *in*determinate which one it is (Torza, 2020); and some may believe, as suggested (but not endorsed) by McSweeney (2019), that there is a non-arbitrary, non-redundant set of fundamental logical particles, but that *which* set that is, is currently unknown to us. Monroy Pérez (2022) presents a new proposal to solve the riddle. I will now explain his proposal, and then I will present four problems, of increasing gravity, for it.

2 The Ramseyan Proposal

The proposal that we will examine here is a systematic development of an idea by Ramsey, worth quoting at length:

> The conclusion of a formal inference must, I feel, be in some sense contained in the premisses and not something new; I cannot believe that from one fact, *e.g.* that a thing is red, it should be possible to infer an infinite number of different facts, such as that it is not not-red, and that it is both red and not not-red. These, I should say, are simply the same fact expressed by other words; nor is it inevitable

[1] The problem may be phrased more generally, in terms of *deductively closed theories*—*cf.* Sider (2011), Donaldson (2015), Monroy Pérez (2022, pp. 1087–1090). A more formal definition can be given in terms of the notion of *Morita equivalence*, see Monroy Pérez 2022, pp. 1087–1088. For simplicity, I use the notions of redundant and arbitrary sets of logical particles.

[2] For some reasons in favour of this view, see Monroy Pérez 2022, pp. 1090–1094, see also McSweeney 2019.

> that there should be all these different ways of saying the same thing. We might, for instance, express negation not by inserting a word 'not,' but by writing what we negate upside down. Such a symbolism is only inconvenient because we are not trained to perceive complicated symmetry about a horizontal axis, and if we adopted it we should be rid of the redundant 'not-not,' for the result of negating the sentence 'p' twice would be simply the sentence 'p' itself. (Ramsey, 1927, pp. 161–162)

So, in Ramsey's original thinking, the main objective was to substantiate the idea that facts like Pa, $\neg\neg Pa$ and $Pa \wedge \neg\neg Pa$ are really just one and the same fact. As a more perspicuous expressive device, he proposed to adopt a symbolism where negation is not expressed by a separate symbol or a separate word, but by the horizontal reflection of what is negated.

In Monroy Pérez's implementation of the idea, however, the main objective is to solve the riddle of redundancy. In his way of posing the problem, what we want is to 'account for [logical] vocabulary being elite' (2022, p. 1096), because 'any formulation of a good theory will need to resort to words such as "and", "or", "some", and the like'. As we will see below (§6), I disagree with this manner of posing the problem.

At this point, it is important to be absolutely clear about what we mean with words like 'symbol', 'notation', 'vocabulary' and 'language'. In this paper, these terms refer exclusively to *uninterpreted symbols*—purely syntactic elements devoid of semantic content. As an anonymous referee has noted, some participants of this discussion (*cf.* §6, below), are not always entirely clear whether terms like 'vocabulary' or 'word' refer to purely syntactic elements or as interpreted symbols, which can lead to confusion.

Abstractly, a set of *symbols* (i.e., a *vocabulary*) is any set of distinguishable elements with sufficient cardinality for the task at hand. I take 'notation' as meaning 'language' in the logical sense. A *language* consists of a set of symbols and a set of *formation rules*, which recursively generate sequences of symbols—referred to as *formulae*. These rules may take sequences of symbols or other formulae as inputs. Such a language is not by itself tied to any particular semantical interpretation—although, of course, we may have ulterior semantical motives in investigating a particular syntactical structure.

As all sequences, formulae are mathematical objects. Given the multiple instantiability of mathematical structures in general, a single abstract structure may be realised in different physical forms. For example, the same formula—the mathematical structure—can be *written* or *printed* in various formats. In conventional notation, formulae are typically written left to right; but they could equally be written vertically, diagonally, or in other arrangements; or printed in blue ink, etc. However, in this paper, I am not concerned with the physical manifestations of these structures—logical languages—but rather with their abstract properties.

While the languages in question are abstract logical structures, the *joints* that they are supposed to represent are not: for this debate, the basic assumption is that these joints are part of the metaphysical structure of our concrete reality; further, these joints are the whole point of caring about elite languages (see §6, below). For the moment, however, let us carry on the discussion in the more formal mode in which it has been developing.

Going back to Monroy Pérez's implementation of Ramsey's idea, his plan is to 'achieve our goal by messing with notation' (p. 1906). Specifically, in the proposed Ramseyan signature for classical zeroth-order logic, the only primitive symbol is '$\wedge$', and negation is incorporated through a non-standard formation rule:

For any formula ψ, the negation of ψ is its reflection over the vertical axis: $\scalebox{1}[-1]{$\psi$}$.

Of course, 'ψ' is *deductively and semantically equivalent* to any formula where an even number of negations are put before 'ψ' itself. In the Ramseyan setting of Monroy Pérez, 'ψ' is *also* the same *formula*—the same string of symbols—as each of its evenly-negated versions. This means that the Ramseyan system cannot express doubly-negated, and indeed any evenly-negated, formulae, because (as an anonymous reviewer accurately commented) the result of twice negating a formula is the very same formula (i.e., it's not two different formulae presented in the same way).[3] This means that the rules of double-negation introduction and elimination become redundant (but no meta-logical properties such as soundness, completeness or decidability are affected).

Because its only primitive symbol is '$\wedge$', the Ramseyan proposal seems to be ahead by simplicity, relative to the traditional setting, at least counting by the mere cardinality of the primitive signature. (One might object that the Ramseyan proposal does not trump all traditional alternatives, such as employing the Scheffer stroke, but see §3, below.)

The thought is that we can define the other connectives from here. Using the traditional notation plus '$\Longleftrightarrow$' for logical equivalence, let us remember the De Morgan equivalences:

$$\begin{aligned} p \vee q &\Longleftrightarrow \neg(\neg p \wedge \neg q), \\ p \wedge q &\Longleftrightarrow \neg(\neg p \vee \neg q), \end{aligned}$$

which connect disjunction and conjunction as duals of each other, and are customarily used to introduce one as definable from the other.

Now, using the Ramseyan notation, Monroy Pérez argues that '$p \vee q$' turns out to be not only equivalent but the very same string of symbols as its proposed *definiens*. Let us see the sequence of transformations, using '$\doteq$' for formula identity and the usual symbol for negation in the intermediary steps of the translation:

$$\begin{aligned} p \vee q &\Longleftrightarrow \neg(\neg p \wedge \neg q) \\ &\Longleftrightarrow \neg(\scalebox{1}[-1]{p} \wedge \scalebox{1}[-1]{q}) \\ &\doteq p \vee q \end{aligned} \tag{1}$$

[3] At this point Monroy Pérez commented about his proposed notation that the 'distinction between atomic and complex formulae is fuzzy', because 'It is not clear whether p is atomic or complex, as it might be the result of applying negation twice' (p. 1104). But this is incorrect, and on two fronts. *First*, it *is* clear whether p is atomic or complex, even if it is the result of applying negation twice: p is atomic, and applying negation twice results in p again, i.e., in the original atomic formula. Second, if this *were* not the case, the result would not be 'fuzzyness' (there are no borderline cases), but *ambiguity*, as 'p' would be ambiguous between p and *not-not-p*—and, indeed, between any odd, finite number of negations applied to p. The resulting *infinite ambiguity* would be disastrous for this notational system. Luckily, however, no such ambiguity results: rather, the Ramseyan system is less expressively powerful than the usual notation.

Monroy Pérez argues that this means that the Ramseyan proposal for the signature of zeroth-order logic is a singleton: its only element is '$\vee$', i.e, '$\wedge$'. 'The only difference', writes Monroy Pérez (p. 1100) 'is the orientation of the symbols, but this sort of orientation is not significant in the subpropositional level on my approach; it is only significant for formulae'. *Just as you keep your identity if you stand on your head, we have one and the same symbol here*: '$\vee$', i.e, '$\wedge$'.

For the quantifiers, if we use '$\bigwedge$' for the universal quantifier and '$\bigvee$' for the existential quantifier, we can see, using again the corresponding De Morgan's law, that '$\bigvee x\psi$' turns out to be not only logically equivalent but the same formula as its proposed *definiens*—which, in the usual presentation with these symbols, would be '$\neg\left(\bigwedge x\neg(\psi)\right)$'—like so:[4]

$$\bigvee x\psi \;\doteq\; (\bigvee x\psi)$$

According to Monroy Pérez, this means that 'there is no difference between the sets $\{\wedge, \bigwedge\}$ and $\{\vee, \bigvee\}$' (p. 1100). That is, these identities are assumed to hold:

$$\wedge = \vee \tag{2}$$

$$\bigwedge = \bigvee \tag{3}$$

Again, the orientation of symbols is treated as irrelevant to their identity within this framework, so *we have one symbol in each case*. If so, there is no choice to be made between $\{\wedge, \bigwedge\}$ and $\{\vee, \bigvee\}$: 'They are the same set' (*ibid.*) Without a choice to be made, there is no arbitrariness involved, and no danger of redundancy. We have our privileged set of logical particles, it seems: it is $\{\wedge, \bigwedge\}$, i.e., $\{\vee, \bigvee\}$.

We have seen a dilemma in fundamental metaphysics evaporate through an ingenious identification of symbols. This seems too good to be true. I will now argue that *it is* too good to be true.

3 First Problem: Remaining Arbitrariness

We may think that there's no need to even introduce Ramsey's suggestion, as we *already* have a signature for classical zeroth-order logic that consists of a single symbol: *the Scheffer stroke*, '$\uparrow$', the NAND operation, which is equivalent to the negation of the conjunction in the usual notation:

$$(p \uparrow q) \Longleftrightarrow \neg(p \wedge q)$$

As can easily be seen, the Scheffer stroke is functionally complete.[5]

[4] Let us ignore the very minor fact that when the font used is a serif typeface, 'x' and its horizontal reflection do not quite look identical: 'x'. The book of the world may have to be written in sans-serif!

[5] I thank two anonymous referees for suggesting that I include this discussion.

Unfortunately, however, the existence of the Schefferian signature doesn't help us to evade arbitrariness. This is because its dual, *the Peirce arrow* or NOR operation, '$\downarrow$', which is equivalent to the negation of the disjunction in the usual notation:

$$(p \downarrow q) \Longleftrightarrow \neg(p \vee q),$$

is also functionally complete. So, both the Peircean and the Schefferian signatures are (i) singletons, and (ii) functionally complete. Positing that one rather than the other gives the fundamental zeroth-order logical constant is, therefore, arbitrary.

While neither solves the problem of arbitrariness—which the Ramseyan suggestion is intended to solve—each of the Peircean and the Schefferian signatures are as simple as the Ramseyan idea (all three being singletons) *but* do not introduce any new rule for negation, which the Ramseyan idea does. So each of the Peircean and the Schefferian signatures could be argued to be simpler, in the final tally, than this new proposal.

But I don't think that this settles the issue. This is because the Ramseyan proposal is more *powerful*, in two related senses, than the proposal to take either of the other signatures as fundamental.[6]

First, because, as we saw, the Ramseyan idea generalises beyond zeroth-order logic, while the proposal to take either of the Schefferian or Peircean signatures does not.[7]

Second, because using the Ramseyan strategy of writing the negation of a formula as its horizontal reflection, and given the duality of '$\downarrow$' and '$\uparrow$', so that either is definable in terms of the other, we can 'identify' them just as we did with '$\vee$' and '$\wedge$' in (1). That is, suppose we have '$\downarrow$' as our primitive symbol. Then, we can define any formula of the form of '$p \uparrow q$' in terms of a formula only containing '$\downarrow$' (and vice versa, of course). Then, using the Ramseyan idea but applied not to the signature $\{\wedge\}$ but to the signature $\{\downarrow\}$, we can use a translation such as (1) to argue that '$\uparrow$' is really just '$\downarrow$', but flipped — and remember that in this setting, the identity of a symbol is preserved under horizontal reflection.[8] This means that the Ramseyan proposal *applied to the Peircean and Schefferian signatures* solves the problem of arbitrariness between those two signatures as candidates for the fundamental language (see, Monroy Pérez, 2022, pp. 1101-1102).

But this takes us to the real problem. With the Ramseyan proposal, $\{\wedge\} = \{\vee\}$ and $\{\downarrow\} = \{\uparrow\}$. However, this introduces *another* redundancy-arbitrariness dilemma, now with respect to $\{\wedge\}$ and $\{\downarrow\}$, as the Ramseyan proposal offers no non-arbitrary basis

[6] As we will see below (§5), the Ramseyan proposal is but a case of a general strategy, what I call *the involutory strategy* The real claim is that any case of the involutory strategy is more powerful, for reasons analogous to the ones immediately explained.

[7] See Monroy Pérez 2022, p. 1098, for his claim that it also generalises to classical modal logic. I am less convinced by this claim, but I won't address it directly here, as my objections here and below apply to the Ramseyan framework whether or not it generalises to modal logic.

[8] Consider the translation (an analogous one exists if we assume '$\downarrow$' as our primitive, of course):

$$\begin{aligned} p \downarrow q &\Longleftrightarrow [(p \uparrow p) \uparrow (q \uparrow q)] \uparrow [(p \uparrow p) \uparrow (q \uparrow q)] \\ &\Longleftrightarrow (\reflectbox{p} \uparrow \reflectbox{q}) \uparrow (\reflectbox{p} \uparrow \reflectbox{q}) \\ &\doteq p \downarrow q \end{aligned}$$

for choosing between these as our fundamental signature, while taking both would result in redundancy.

Monroy Pérez (2022, pp. 1101-1102) expresses to be 'tempted to claim that '$\wedge$', '$\vee$', '$\uparrow$' and '$\downarrow$' can be shown to be the same connective', but no matter the argument, he should resist the temptation: no amount of horizontal reflections—the fundamental novelty introduced by the Ramseyan proposal—will turn '$\vee$' into '$\uparrow$', so if 'connective' here means *symbol that represents the truth-function*, the claim is seen to be mistaken.[9] If, rather, 'connective' here means *the truth-function*, then, the claim is again seen to be mistaken, as $\vee(T, T) = T = \wedge(T, T)$, while $\uparrow (T, T) = F = \downarrow (T, T)$. Nor is any other plausible criteria for connective-identity satisfied here. We can *define* either in terms of the other, of course (given the Ramseyan negation), but this does not suffice to identify any connectives: truth functions are defined by their input–output behavior; therefore, even if AND can be defined in terms of OR, they remain distinct functions. Indeed, if definability (*per impossibile*) sufficed to identify truth functions, we would have no arbitrariness problem to begin with: AND and OR are already interdefinable in the traditional setting.

So, there remains a problem of arbitrariness for zeroth-order logic that the Ramseyan proposal does not solve: assuming the Ramseyan proposal for the formation rule for negation, the signatures $\{\wedge\}$ (i.e., $\{\vee\}$) and $\{\downarrow\}$ (i.e., $\{\uparrow\}$) are equally simple and truth-functionally complete, with no other relevant distinguishing feature. If so, it is arbitrary to postulate one rather than the other as the signature of zeroth-order part of the fundamental language. Sadly, however, this is the least of the problems for the Ramseyan proposal. As I now explain, it can *itself* be argued to be arbitrarily chosen among structurally similar ideas.

4 Second Problem: The Revenge Of Arbitrariness

The Ramseyan proposal is not the only one of its kind, so even if it solved the problem of arbitrariness, there would be *another* problem of arbitrariness. (Let us focus on the propositional case for now; analogous arguments can be made for the first-order and modal extensions).

We have the following, alternative signature for propositional classical logic: $\Sigma = \{<\}$, and just as in Ramseyan notation, negation is not a symbol but is expressed *via* our formation rules. In particular, negation flips (sub)formulae *vertically* now; so as to track the relevant symmetries, our formulae are now not row, but *column* vectors. Then, this:

$$\begin{pmatrix} p \\ < \\ q \end{pmatrix}$$

[9] In p. 1102, Monroy Pérez notes that he needs to (i) remove part of the symbol '$\uparrow$'—which of course leaves us with *another* symbol—and (ii) distinguish true from false formulae even before we get to the model theory (he uses a dent on the imagined sheets of a supposed 'book of the world' that distinguishes the side that contains truths from the side that contains falsehoods). This is no longer merely flipping symbols to express negation, of course, so this is no longer the Ramseyan proposal, but an under-specified extension of it.

is intended to mean 'p or q'. Using our De Morgan definition for disjunction, we can also see that '$p \vee q$' turns out to be not only equivalent but the very same string of symbols as its *definiens*:

$$
\begin{aligned}
\begin{pmatrix} p \\ < \\ q \end{pmatrix} &\Longleftrightarrow \neg \begin{pmatrix} \neg p \\ > \\ \neg q \end{pmatrix} \\
&\Longleftrightarrow \neg \begin{pmatrix} \scalebox{1}[-1]{q} \\ > \\ \scalebox{1}[-1]{p} \end{pmatrix} \\
&\doteq \begin{pmatrix} p \\ < \\ q \end{pmatrix}
\end{aligned}
\tag{4}
$$

As we see, my signature for propositional classical logic, $S = \{<\}$, also: (i) is a singleton, (ii) is such that the definition of a disjunction (or, alternatively, conjunction) is the very same formula as its *definiens*, and (iii) therefore, solves the riddle of redundancy just as much as the Ramseyan proposal does. Of course, in my proposal, formulae are column vectors, while the Ramseyan proposal preserves the row vector form of the traditional notation; but the syntactic complexity and the commitments remain the same, so this is of nil logical and metaphysical relevance.

Now we have to choose between two alternative fundamental signatures: $\{\vee\}$ and $\{<\}$, each with their corresponding formation rules for negation. They are equally simple and equivalent in the other relevant aspects. And this means that choosing between the Ramseyan proposal and mine is, ultimately, *arbitrary*. And we should not adopt a signature containing both my and the Ramseyan primitives, as it would be clearly *redundant*. The fundamental dilemma between arbitrariness and redundancy has not been solved.

Going to the essence of the matter, the rule for negation need not be a reflection in the plane. We can see this because both the Ramseyan and my new proposal are cases of a more general idea. First, note that classical negation is an *involution*: a self-inverse operation. Formally, an involution is a function f such that $f(f(x)) = x$ (for any x in its domain), so that applying f any even number of times leaves the entry intact. Then, the essence of the general idea is to represent negation by means of an involutory transformation: we have seen it represented by means of vertical and horizontal reflections, but the only limit to this is really our imagination. For example, we might abstractly 'tag' formulae with colours, so that f might be the operation *Tag with the complementary colour* (relative to, say, the RGB colour model), so if 'p' is tagged with blue, '$\neg p$' would be 'p' but tagged with yellow, and then '$\neg\neg p$' would be 'p' tagged with blue again. In this way, the involution f transforms the signature in some specified space such that, applying it any even number of times leaves us with exactly the original symbol. In the Ramseyan proposal, f is an horizontal reflection in the plane; in my examples, f is a vertical reflection or a transformation in a colour space. Indeed, Ramsey's notation is analogous to the notation that apparently was used earlier by De Morgan, who expresses negation not by flipping the symbols but by changing between uppercase and lowercase letters (*cf*. Schlimm, 2025).[10] Again, once we allow ourselves to 'mess with notation', the possibilities are only bounded by our imagination: perhaps our set of symbols consists of sound waves (symbols need not be inscriptions!), f inverts the phase of each symbol, and then '$\neg\neg p$' is the original sound wave. In each such case, of course, we would have to adjust our formation rules—but so what? We also had to adjust them in the Ramseyan proposal.

As we see, once we allow symbolic devices to solve metaphysical riddles, the floodgates are open, and the overdetermination of theories is reinstated—but now the question is no longer which logical constant represents the metaphysically fundamental logical structure, but rather which theory about *which formation rules should govern the fundamental language* is correct.

Could it be argued that, still, the Ramseyan proposal is simpler? Not quite. For in one case, the rule that negates is given by reflection. In the other, the rule that negates is acting on colour tag. Each proposal has a single rule for negation. What about horizontal vs. vertical reflection, or about reflection vs. colour? Not any particular one stands out as the simplest. Both relative position and colour are required for a printed version of our logical formulae. Neither proposal requires more structure than what is already in place as things stand. True, in the case of sound waves, there may be some inconvenience, but so what? There is no intrinsic reason to privilege written over auditory representations of logical structures: why *write* the book of the world *instead of singing it*?

The fundamental reason for this problem lies in what is the absolute minimal structure needed by the *general involutory strategy*. It is: (i) a *vocabulary* for zeroth-order logic, which is simply any set of sufficient cardinality, (ii) a set of *propositional letters*, which is any countably infinite set, and (iii) the formation rule for negation being an involutory operation of the *symbols* (i.e., vocabulary plus propositional letters) and formulae (i.e., finite sequences of symbols delivered recursively by the formation rules) relative to some specified space. The Ramseyan proposal is a particular case of this strategy. But the abstract structures can be realised in any of an indefinite number of ways, each with no salient feature relative to their peers, which creates a further problem of arbitrariness.

5 Third Problem: Even Simpler Solutions

We have seen that the Ramseyan proposal is a special case of the general idea of coding negation in terms of an involutory rule of formation and making the needed adjustments elsewhere. But it gets worse.

The general involutory strategy—and, of course, the Ramseyan case—is not the simplest. Remember that the Ramseyan proposal for classical first-order logic is this signature:

$$\left\{\vee, \bigvee\right\}$$

[10] I thank an anonymous reviewer for this pointer.

However, I now argue that I can make do with a singleton, so that an even simpler signature is available. Given that simpler theories are preferable to less simple theories, this is not a case of the riddle of redundancy, as it is not arbitrary to pick the simpler theory. But it does show, however, that the Ramseyan proposal is not even a competitor. Even more, it leads us to reconsider the rules of the game—which is where I think the deepest problem lies.

Consider this signature:

$$\{<\}$$

Then, I express some classical notions in this way:

$$\begin{pmatrix} p \\ < \\ q \end{pmatrix} \Longleftrightarrow p \text{ or } q \qquad \begin{pmatrix} p \\ > \\ q \end{pmatrix} \Longleftrightarrow p \text{ and } q$$

$$\begin{pmatrix} < \\ x \\ p \end{pmatrix} \Longleftrightarrow \text{ for some } x,\, p \qquad \begin{pmatrix} > \\ x \\ p \end{pmatrix} \Longleftrightarrow \text{ for all } x,\, p$$

In my language, the primitive symbol represents a propositional constant or a quantifier depending on whether or not it occurs infixed between two formulae or prefixed to a variable. Just as negation is a vertical reflection, the formation rules also exploit the difference between infix and prefix positions. And note that there is no obvious ambiguity here. For example, what in the traditional notation would be expressed by '$\neg(\forall x\,((\neg Fx) \vee (Gx)))$', in my notation is expressed as follows (again, I use the usual symbol for 'not' in the intermediary steps of the translation):

$$\begin{aligned} \neg(\forall x\,((\neg Fx) \vee (Gx))) &\Longleftrightarrow \neg \begin{pmatrix} > \\ x \\ \begin{pmatrix} \neg(Fx) \\ < \\ Gx \end{pmatrix} \end{pmatrix} \Longleftrightarrow \neg \begin{pmatrix} > \\ x \\ \begin{pmatrix} \scalebox{1}[-1]{Fx} \\ < \\ Gx \end{pmatrix} \end{pmatrix} \\ &\Longleftrightarrow \begin{pmatrix} < \\ \scalebox{1}[-1]{x} \\ \begin{pmatrix} Fx \\ > \\ \scalebox{1}[-1]{Gx} \end{pmatrix} \end{pmatrix} \Longleftrightarrow \exists x(Fx \wedge \neg Gx) \end{aligned} \tag{5}$$

There is no additional structure here. The distinction between infix and prefix positions exists in both the traditional and Ramseyan proposals, and using vertical instead of horizontal arrangements reflect practical rather than substantive differences—my proposal only exploits what is already given.[11] So, this signature is more economical

[11] Indeed, we may combine the very same infix/prefix trick with the rule that transforms through a horizontal flip, the important point for now is that we really only need a single primitive symbol.

and therefore preferable to the Ramseyan proposal. Again, once we allow 'messing with notation' as a way to solve metaphysical riddles, the possibilities are only bounded by our imagination.[12]

6 The Fourth and Most Fundamental Problem

In the Ramseyan proposal and in all other cases of the involutory strategy, '$\vee$' and '$\wedge$' will result in the same symbol: the same uninterpreted element of the set that constitutes our abstract language, just transformed by the involutory operation. However, *such a symbol needs to represent different truth functions in different contexts*. This is because the whole point of the strategy of 'messing with notation' was using a single symbol to represent a connective *and its dual*. And we need *both* truth-functions for the expressive completeness of logic.

In Monroy Pérez's proposal, vee ('$\vee$') and wedge ('$\wedge$') are considered the same symbol, being rotations of a basic geometric shape. This symbol represents different truth functions depending on its rotation. This distinction is crucial to understanding why the equivalence of the symbols at a syntactic level does not translate into equivalence at the semantic level, so let's illustrate it with an analogy.

Think of a square: $\square$, and a 45° rotation of it in whatever direction: $\diamondsuit$. Geometrically, we have *the same shape* (same angles and side lengths), but transformed under a rotational symmetry (we don't shear or deform the shape in any other way, as with $\Diamond$). However, the context in which the shape appears may imbue each rotational state with a distinct semantic value: say, '$\square$' represents message M in a communication system, while '$\diamondsuit$' represents message N. Thus, while the shape remains the same, its rotational state determines the message it conveys. Similarly, in the Ramseyan proposal, pointing upwards, the symbol represents a certain truth function f, one such that $f(T, F) = F$. Pointing downwards, the symbol represents *another* truth function g, such that: $g(T, F) = T$.

To compare, consider what happens completely outside of the involutory strategy. Consider again the Schefferian signature, $\{\uparrow\}$, with the usual rules (negation being '$\neg$', as usual). Of course, the intended interpretation of the symbol '$\uparrow$' is the NAND truth function, so that it takes (T, T) to F. As we know, the Schefferian signature is truth-functionally complete, which means that every possible truth function can be expressed using only '$\uparrow$'. Now, in this notation we can 'simulate' negation (i.e., the truth function $n(T) = F, n(F) = T$) because we can define:

$$n(x) = \uparrow (x, x),$$

where x is a truth value. So, although NAND is a *binary* operation, it can simulate a *unary* operation (negation), because with our definition, whenever a single formula is true, we have a function that delivers false and vice versa: even if NAND is binary, we can define an operation that effectively toggles *single* truth values. And equivalently

[12] Could we go even further? We might imagine a system with *no* primitive symbols for the logical particles, that goes by with different colours or positions of the non-logical symbols. In such a case, however, a gain in economy would leave us with no obvious choice for our fundamental logical language.

for NOR, of course—in general, binary operations can sometimes simulate unary operations when both inputs are the same.

However, going back to cases of the involutory strategy, they do not *simulate* disjunction in terms of conjunction, nor vice versa. Instead, we have a *single symbol*, but it represents the conjunction function *in some contexts*, and it represents the disjunction function *in other contexts*. In particular, in the Ramseyan proposal, '$(p \wedge q)$' is true iff both p and q are true, and '$(p \vee q)$' is true iff either p or q are true, and '$\vee$' can be defined by flipping from '$\wedge$'.[13]

This takes us to the important point. As we have seen, even if the syntactic machinery allows us to say that we have the same symbol, only positioned one way in some circumstances and another way in other circumstances, it does *not* allow us to say that *what the symbol means* remains the same across flips.

But the problem was never about symbols. It was about *what is the fundamental logical joint*: is it the one represented by conjunction, disjunction, NAND, NOR, or perhaps some other connective? The question is *which* of the logical particles the symbols can represent *is* the fundamental one. Let me explain.

'The language of the book of the world' is, of course, a descendant of Lewis' proposal that there are certain special predicates: those that refer to the *natural properties* (Lewis, 1983, 1986). Natural properties are a very important part of Lewis' metaphysical system, as is widely known (cf. Dorr & Hawthorne, 2013).

To get to 'the book of the world' metaphor from here, we must generalise on (at least) two fronts. First, we must separate naturalness from the debate on the reality of properties — so that naturalness is a legitimate posit even if nominalism turns out to be true. To do so, naturalness is to be characterised as an *operator*, '$\mathcal{N}$', attaching to predicates to form a sentence. Then, for example, if it is true that $\mathcal{N}(P)$, then 'P' marks a distinction between objects such that those that are P are objectively similar, the distinction between being P and being not-P forms part of the supervenience base for the rest of the distinctions, and so on — independently of whether or not there is a *property* referred to by 'P'. Second, we 'go beyond the predicate' because 'The crucial expressions in ontology, logic, and modality [...] are quantifiers and operators, not predicates' (Sider, 2011, p. 8.). Then, we arrive at (something very much like) Sider's concept of *structure*, which allows us 'to ask, of expressions of any grammatical category, whether they carve at the joints' (*ibid*.) We have generalised from predicates to other grammatical categories. And going from natural *properties*

[13] A reviewer kindly suggested that, because we can take the truth condition for conjunction, do a Ramseyan flip on all the object-language formulae occurring in it, and get as a result a correct statement of truth conditions for '$\vee$' (effectively the equivalent of '$\neg(p \vee q)$' is true iff '$\neg p$' is true and '$\neg q$' is true), this could indicate that there is some sense in which the same truth condition seems to define the meanings of '$\vee$' and '$\wedge$', in this setting. Further, this would be a reason to think that the symbol does express the same truth function in either orientation. Now, it is true that flipping the components of the truth condition for '$\wedge$' effectively yields the truth condition for '$\vee$', consistently with classical duality. *However* this flipping does not imply that '$\wedge$' and '$\vee$' represent the same truth function: it only shows that their truth conditions are related by negation (flipping). But truth functions are defined by their input–output behaviour, which as we have seen, is different: one is a function f such that $f(T, F) = F$; another is a function g such that: $g(T, F) = T$. So, the Ramseyan flipping rule allows the conjunction and disjunction symbols to share a syntactic symbol, but their truth functions remain distinct and context-dependent.

to *predicates* was important, in the first place, to let the question of naturalness be independent from the debate on Platonism.

While aiming for neutrality in the metaphysics of properties is a fair concern, this framing also invites an overly linguistic conception of the matter. For example,

> On my subpropositional approach […] the locus of joint-carving is (to put it linguistically) the word; it is words that do or don't carve at the joints. [Sider, 2011, p. 220]

Other philosophers also present the view in terms that are overly linguistic. Consider, for example:

> Elitists distinguish the *elite*, upper-class words from lower-class *plebeian* ones. […] the elite words are somehow particularly well-fitted to the world's intrinsic, fundamental structure […] When using the plebeian terms, however, we can at best describe the world as it appears to *us* [Donaldson, 2015, pp. 1051-1052; for more examples see Warren, 2016, p. 2418; McSweeney, 2019, p. 118]

I believe that these ways of framing the matter are dangerously confusing, inviting the notion that elitism is a matter of choosing the correct language.

However, Sider himself noted the dangers of conceiving of the issue in terms of languages:

> The epistemology proposed […] instructs us to search for the most explanatory pair $\langle I, T_I \rangle$ of ideology I, and theory T_I in terms of I. A natural concern is that there will be no unique most explanatory pair. My response to the concern is that when pairs are tied, we should be agnostic which pair is correct […] But—so the worry goes—this will lead to misplaced agnosticism if a new ideology-theory pair can be produced by changing to Spanish, red ink, or boldface. [Sider, 2011, p. 221]

He then suggested a conception of the notion of *language* tailored for logical elitism:

> The answer to the worry is that these changes needn't produce new pairs. […] the ideology-theory pairs must be formulated in some language, and the sentences of this language will not be individuated so finely as by language family, font, or ink color [*ibid*.]

Of course, I agree with the underlying sentiment; but then again, using 'language' or 'sentence' in a way such that 'red', 'rojo', 'rouge' and 'rot' are all words *from the same 'language'* is a misnomer: such a notion is much too wide to be counted as a notion *of a language*, and as a conception of sentence-identity, it invites unfruitful questions like whether or not it makes '$p \wedge q$' and '$p \vee q$' the same sentence.

We may try a quick fix to this problem by invoking synonymity. Donaldson (2015, p. 1055) suggests a Siderian epistemology of eliteness thusly (my emphasis):

> we are justified in believing that a term is elite just in case we are justified in believing that either it *or a synonym* occurs in all of the most virtuous total theories.

However, two theories may be *non-synonymous* and still represent the same underlying structure: synonymity is not necessary. Consider the Lagrangian and Hamiltonian formulations of classical mechanics: the Euler-Lagrange equation contains 'words' (interpreted symbols) non-synonymous to any 'word' in Hamilton's equations—like L, the Lagrangian function. Still, it has been argued that both formulations represent, ultimately, a common deep structure: symplectic structure (Belot, 2007; Thébault, 2016). In quantum mechanics, as well, matrix and wave formulations represent states within a common underlying structure: Hilbert space. Further, these structures are underpinned by *deeper* or more *abstract* structures, characterised by the invariants identified through symmetries in group theory (the fundamentality of these symmetries relative to other physical structures remains debated, e.g., French, 2014). But if we focus on the *words*, matrix and wave mechanics (on the quantum side) and Lagrangian and Hamiltonian mechanics (on the classical side) would have to be counted as *competitors* for supplying us with the quantum and classical joints of nature (respectively)—and indeed, some have taken that view (e.g. North, 2009; Curiel, 2014, for the classical case). The linguistic framework might misleadingly suggest that these scientifically equivalent pairs are competing candidates for joint-carving ideologies—which can be argued to be problematic for Sider's intended naturalism (Warren, 2016, p. 2423).

So, what is important is not sameness of symbols and not even sameness of meaning, but *sameness of represented structure*. What should take priority in the methodology of elitist metaphysics is the question of what are the joints, not what words are used to mean them. Then—going all the way back to Lewis—while aiming for neutrality in different first-order ontological questions is a fair concern, we must remind ourselves that this was always a *schema*. In the end, the metaphysical structure of the world—if such there be—has nothing to do with any *words*, any *grammar*, or any *symbols*. We do want a *good* (by elitist standards) theory to *represent* the joints of nature; but this is secondary, and if we prioritise it, it obscures the fact that elitism should be thought of primarily as a *metaphysical* thesis about *joints*: minimally, the thesis that *there are* such and such discernible joints in nature.[14] The question whether or not, and how, and to what extent, any given representational apparatus captures such joints, is a thesis of epistemology, semantics, or in general of a theory of representation.

So, we should primarily and firstly (in a methodological sense) query about *what are the joints*, and then evaluate (for the purposes of this discussion) our representational

[14] Note that this is *compatible* with Sider's (2011, pp. 92) clarification that:

> To say $\mathcal{S}$(and) is not to say something about an alleged object Conjunction. It is not to say anything about any thing at all. It is nevertheless to say something true, something objective, something about reality. Nowhere is it written in stone that all facts must be entity-involving.

I have *not* assumed that joints are objects in the logical category of individuals—or properties of them, or of any category. If 'reification' means—as it most usually does—*to take something as real*, in this sense, Sider *has* to reify joints. Without joints, the whole project of Lewisian fundamentalist meta-metaphysics loses its aim, because the metaphysical structure of the world is *given* by those joints: we have to assume that *joints are real*—that they are something answering to the (purported) truth of '$\mathcal{S}$(and)', which does not float free from reality. This does not require there being an *object*—e.g. an operation—that is being claimed to be the joint: what we cannot coherently evade is commitment *to the joints*. Without joints, there is no metaphysical structure, and without metaphysical structure, the idea that some ideology stands for it loses any significance.

devices in terms of their relations to such joints. Using a fairly standard notion of a language, we accept that a fundamental theory—'the book of the world'—can be written in more than one language—in this standard, non-problematic sense—for the trivial reason that different languages can be used to represent the same joints; we also accept that two *different* theories may represent the same joint, being *equivalent* in a deeper manner (e.g., by shared symmetries).

We may then individuate candidates for *joints representations* as equivalence classes under the relation of *having the same joint as intended represented structure* (given a fixed interpretation and logic). For a toy case, consider these words and their intended interpretation: 'red', 'rojo', 'rouge' and 'rot' all belong to the same class (but not 'blue' nor 'azul'), $[\![r]\!]$, as they represent the same purported joint: the joint marking the distinction between the things that are red and the ones that are not (and analogously for pieces of different languages and signatures like '$\wedge$', '$\cdot$', 'and', 'y' and 'et', which all belong to $[\![\text{conjunction}]\!]$, but not '$\vee$'). As Lewis wanted, such a joint's nature can turn out to be one among various conceivable possibilities: perhaps a Platonic universal, perhaps a nominalistic primitive difference. We could then let the accepted scientific standards—at least those that are not clearly motivated by merely instrumentalist or practical aims—filter out the candidates for our best theory, from which we would extract our candidates for joint representations.

By accepting the unproblematic notion that many fundamental languages and theories are possible, we emphasise our primary interest: *that they represent the joints*. Doing so allows us to leave behind the idea that elitism is essentially about words and languages; in particular, logical elitism is about the reality and nature of the logical joints—whether we use '$\vee$' or 'or' to represent one of them is of very secondary interest.

Given the foregoing, the dilemma between arbitrariness and redundancy is not a dilemma about *languages*—but about *which logical joint is the fundamental one*. For the case of zeroth-order logic, the *conjunctive* and the *disjunctive joints* are contenders, and there may be more; for the case of first-order logic, the main contenders are the *universal* and the *existential joints*, but again there may be more. And these joints can be specified independently from languages and notations, simply because they can be univocally specified in each of indefinitely many different representational systems.

But then, it does not really matter whether we can use—by means of a sophisticated flipping technology, or by using colours and positions to our liking—a single symbol to represent both the existential and the universal quantifiers: it does not really matter that $\{\wedge, \bigwedge\} = \{\vee, \bigvee\}$. What *really* matters is that we answer questions like: *Which among the existential and the universal logical joints is the fundamental one?* And, sadly, no notational technology, no matter how ingeniously designed, is going to help answering them.

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